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RELIABILITY BASED ANALYSIS OF BEARING CAPACITY OF FOOTINGS ON REPLACED SOIL

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ABSTRACT
Geotechnical engineering designs depend on soil parameters and theoretical formulas which lead to inaccurate results. The lack of accuracy is covered by the factor of safety. The complementary solution is the reliability analysis which gives an index to choose the proper factor of safety to make the design safer and more economic. In this study, a procedure for carrying out reliability analysis of bearing capacity of foundation resting on soil improved by soil replacement with different dimensions of replacement (8 models) is described. The procedure requires definition of standard deviation of the undrained shear strength, angle of internal friction and the unit weight of the stone material and the surrounding soil.

The procedure is an extension of the point estimate method in which the expected values of the standard deviation of the capacity and demand functions are calculated. The probability of failure, the reliability, central factor of safety and reliability index are calculated as appropriate.

It was concluded The reliability and the probability of failure depend mainly on the approach used in the estimation of bearing capacity, a number of equations were derived in the literature to estimate the bearing capacity but the degree of conservation for each one is different from one to another. In some cases analysed in this study, the probability of failure was found to be less than 1% depending on the value of reliability index obtained from reliability tables which are always greater than 2.2 based on standard normal distribution.

INTRODUCTION
Applications of reliability in geotechnical engineering have increased in recent years remarkably. The conventional design in geotechnical engineering should consider the calculation of a reasonable factor of safety. However, due to the large uncertainty resulting from in-situ soil variability, even in homogeneous soils, it may not always represent a realistic situation. The effect of variability in soil properties cannot be efficiently modeled in such an analysis. For these cases, the use of reliability analysis to model ground uncertainties. Modern building codes are based on Load and Resistance Factor Design (LRFD) approaches, which are in turn based on reliability methods. These techniques are now being introduced into such areas as pile design for highway structures.

The sources of uncertainty are unavoidable and they come from the following (Bowles, 1996):
1- The incomplete knowledge of the subsoil conditions.
2- Inherent variability in soil parameters.
3- Lack of control over environmental changes after construction.
4- The accuracy of the theoretical or empirical methods for calculating bearing capacity.
5- Predication of the applied loads such as dead loads, live loads, wind loads, earthquake, etc.

Accordingly, the design of foundation is uncertain, in general, variability and randomness cause a difficulty in selecting the suitable design parameter.

During the last few decades, numerous remarks were raised against the factor of safety, as many authors see disadvantages in disregarding the reliability of the applied data and the risk reflecting the economic background. In other word, the empirical choice of a certain value of a safety factor does not convey the safety quantitatively and its effect can be neglected by presence of large uncertainties in the design environment.

Kenny and Andrawes (1997) presented a theoretical model for the case of footings resting on a sand layer overlying clay deposit. Model tests were carried out in the laboratory to evaluate the stress – settlement relationship of the sand alone, clay subgrade alone, and the sand overlying clay. The stress – settlement relationships for all tests were then presented in non – dimensional form, and the results of this investigation are compared with experimental data reported by other researchers.

Fattah et al. (2003) made a trial to improve the bearing capacity of a strip footing resting on soft clay using a trench of sand. In addition to that, geogrid reinforcement is placed in a horizontal position or lining the channel. This principle is similar to the principle of using stone columns to improve the bearing capacity of soft soils. The study showed the possibility of improving the bearing capacity of strip footing resting on soft clay using a sand channel with different inclinations. It was shown that the best improvement can be obtained when the channel slope is 60º. When using geogrid reinforcement in the channel, the best increase in bearing capacity was obtained when making the geogrid lining the channel at an angle of 60º.

Dasaka et al. (2005) investigated the probabilistic analysis of bearing capacity of strip footing resting on cohesionless soil deposit. The calculated factors of safety corresponding to a target reliability index of 3 are 7.3 and 5.5 respectively for simple and advanced probabilistic analysis. These factors of safety are generally considered higher than those adopted in routine foundation designs. The higher values of factors of safety associated with allowable bearing pressure obtained by probabilistic approach clearly demonstrate the importance of uncertainty studies in geotechnical engineering and strongly demands the need to include probabilistic framework in geotechnical engineering design.

Honjo et al. (2011) proposed scheme of a reliability based design (RBD) for practicing geotechnical engineers. The essence of the proposed scheme is the separation of the geotechnical design part from the uncertainty analysis part in geotechnical RBD. In this way, practical engineers are able to perform RBD in a more comfortable way compared to the traditional RBD procedure. Based on the results, some discussions were made to identify the major issues geotechnical RBD is facing. It was concluded that spatial variability of soil properties is only one of the sources of uncertainty. In many design problems, statistical estimation error, design calculation model error and transformation error associated with estimating soil parameters (e.g. friction angle) from the measured quantities (e.g. SPT N-values) have higher uncertainty. It is important to recognize these aspects in developing the geotechnical RBD to next and the higher stage.

The main objective of this work is to show that the factor of safety and the reliability can be used together as a complementary measure of acceptable design and make a comparison between the reliability indices calculated using different equations based on the following conditions using reliability based-design:

1- Cohesion (c_u), angle of internal friction (ϕ) and soil unit weight (γ) are considered to be independent and uncorrelated variables.
2- Footing width and dimensions of the of replacement are set of deterministic variables.

A more logical approach would be considered: A procedure is followed in this paper to investigate the reliability of bearing capacity equation of foundation on soft clay improved by soil replacement based on reliability index rather than conventional factor of safety.

ULTIMATE BEARING CAPACITY OF FOOTING RESTING ON STRATIFIED DEPOSITS OF SOIL

Foundation design must satisfy both strength and serviceability criteria. The soil beneath the foundation must be capable of carrying the structural loads placed upon it without shear failure and consequent settlements being tolerated for the structure it is supporting.

Rupture surfaces are formed in the soil mass upon exceeding a certain stress condition. Hence, bearing capacity is defined as the capacity of the underlying soil and footing to support the loads applied to the ground without undergoing shear failure and without accompanying large settlement (Das, 1999).

The ultimate load failure surface in the soil depends on the shear strength parameters of the soil layers such as; the thickness of the upper layer ; the shape, size and embedment of footing and the ratio of the thickness of the upper layer of the width of the footing . Therefore, it is important to determine the soil profile and to calculate the bearing capacity accordingly.

All the theoretical analyses are based on the assumption that the subsoil is isotropic and homogeneous to a considerable depth. In nature, soil is generally non-homogeneous with mixtures of sand, silt and clay in different proportions. In the analysis, an average profile of such soils is normally considered. However, if soils are found in distinct layers of different compositions and strength characteristics, the assumption of homogeneity to such soils is not strictly valid if the failure surface cuts across boundaries of such layers.

The analysis presented by Das (2007) is limited to a system of two distinct soil layers. For a footing located in the upper layer at a depth D below the ground level, the failure surfaces at ultimate load may either lie completely in the upper layer or may cross the boundary of the two layers. Further, it may come across the upper layer strong and the lower layer weak or vice versa. In either case, a general analysis for (c - □) will be presented and will show the same analysis holds true if the soil layers are any one of the categories belonging to sand or clay. The bearing capacity of a layered system was first analysed by Button (1953) who considered only saturated clay (ϕ= 0).

Later Brown and Meyerhof (1969) showed that the analysis of Button leads to unsafe results. Vesic (1975) analysed the test results of Brown and Meyerhof and others and gave his own solution to the problem. Vesic considered both the types of soil in each layer, that is clay and (c - ϕ) soils. However, confirmations of the validity of the analysis of Vesic and others are not available.

Meyerhof (1974) analysed the two layer system consisting of dense sand on soft clay and loose sand on stiff clay supporting his analysis with some model tests. Again Meyerhof and Hanna (1978) advanced the earlier analysis of Meyerhof (1974) to encompass (c - ϕ) soil and supported their analysis with model tests.

Theoretical estimation of bearing capacity of layered soil
Two equations have been adopted in this paper to estimate the bearing capacity of layered soil which are illustrated below:

Equation 1: (Meyerhof and Hanna, 1978)
As shown in Figure (1), the bearing capacity will be as follow:

$$q_b = c_2 N_{c2} s_{c2} + \gamma_1 (D_f + H) \quad \text{….. (1)}$$

S_{c2} = (1+0.2 B/L) and N_c = 5.14

For ϕ = 0. Therefore

$$q_b = 5.14\, c_2 (1 + 0.2 \tfrac{B}{L}) + \gamma_1 (D_f + H) \quad \text{….. (2)}$$

$$q_t = \gamma_1 D_f N_{q1} s_{q1} + \tfrac{1}{2} \gamma_1 B\, N_{\gamma 1} s_{\gamma 1} \quad \text{….. (3)}$$

Then:

$$q_u = 5.14\, C_2 \left(1 + 0.2 \frac{B}{L}\right) + \frac{\gamma_1 H^2}{B} \left(1 + \frac{2D_f}{H}\right) \left(1 + \frac{B}{L}\right) k_s \tan\phi_1 + \gamma_1 D_f \leq \gamma_1 D_f N_{q1} s_{q1} + \frac{1}{2} \gamma_1 B\, N_{\gamma 1} s_{\gamma 1} \quad \text{….. (4)}$$

The ratio of q_2/q_l may be expressed by:

$$\frac{q_2}{q_1} = \frac{C_2 N_{c2}}{0.5 \gamma_1 B\, N_{\gamma 1}} = \frac{5.14 C_2}{0.5 \gamma_1 B\, N_{\gamma 1}} \quad \text{….. (5)}$$

Where:

N_c, $N_□$	bearing capacity factors for soil,
C_1	cohesion of soil in layer 1,
C_2	cohesion of soil in layer 2,
D_f	depth of footing,
H	thickness of soil layer ,
q_t	bearing capacity of the top soil,
q_b	bearing capacity of the bottom soil,
K_o	coefficient of at-rest earth pressure,

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Ks	coefficients of punching shear resistance under vertical load,
B	width of footing, and
γ	unit weight of soil.

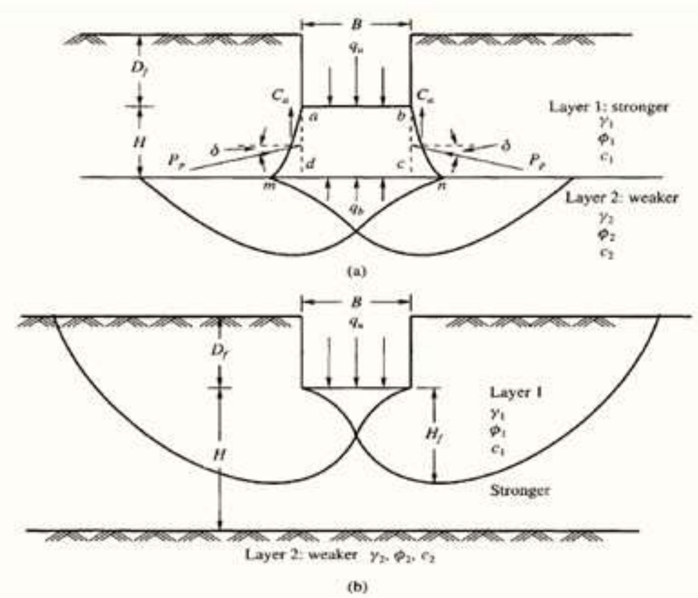

Figure 1 Failure of soil below strip footing under vertical load on strong layer overlying weak deposit (after Meyerhof and Hanna, 1978).

Equation 2: (Madhav and Vitkar, 1978).

Figure (2) shows a continuous rough foundation on a granular trench made in a weak soil extending to a great depth. The width of the trench is W, the width of the foundation is B, and the depth of the trench is H. The width of the trench, W, can be smaller or larger than B. The following equation expressed the ultimate bearing capacity of the foundation with the presence of the trench (Madhav and Vitkar, 1978):

$$q_u = C_2 N_{C(T)} + D_f \gamma_2 N_{q(T)} + \left(\frac{\gamma_2 B}{2}\right) N_{\gamma(T)} \qquad \ldots.(6)$$

Where C_2 = Undrained shear strength of the soft soil, and
$N_{C(T)} N_{q(T)} N_{\gamma(T)}$ = bearing capacity factors with the presence of the trench.

The bearing capacity factors can be found using Figures (I), (II) and (III) in the Appendix.

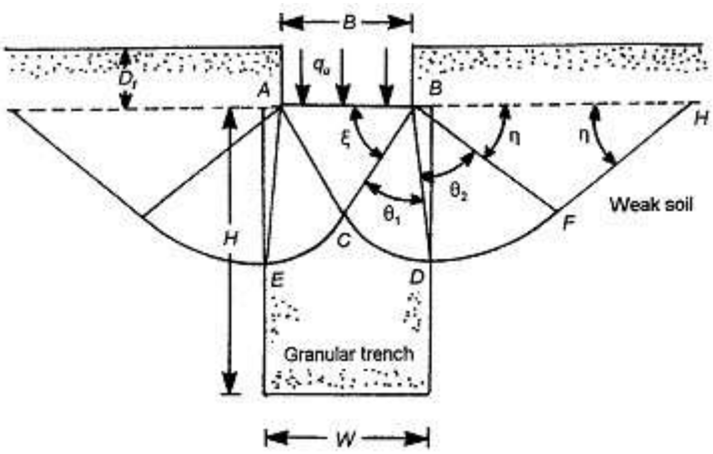

Figure 2 Foundation on a weak soil with a granular trench (Madhav and Vitkar, 1978).

Reliability Analysis of Bearing Capacity of Foundations on Improved Soil

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Simple reliability analyses, involving neither complex theory nor unfamiliar terms, can be used in routine geotechnical engineering practice. These simple reliability analyses require little effort beyond that involved in conventional geotechnical analyses. They provide a means of evaluating the combined effects of uncertainties in the parameters involved in the calculations, and they offer a useful supplement to conventional analyses. The additional parameters needed for the reliability analyses standard deviations of the parameters can be evaluated using the same amount of data and types of correlations that are widely used in geotechnical engineering practice (Duncan and Honorary, 2000).

Selection of Reliability Coefficients

The procedure for carrying out reliability analysis of bearing capacity of foundation resting on soil improved by soil replacement requires definition of standard deviations of the undrained shear strength, angle of internal friction of stone and the unit weight of soil which are taken from Table (1). The coefficient of variation is defined as the ratio of standard deviation to the mean value. The coefficient of variation CoV(x), usually expressed as a percentage:

$$\mathrm{CoV}[x] = \frac{\sigma[x]}{E[x]} * 100\ \% \qquad \text{....... (7)}$$

Table 1: Coefficient of variation of geotechnical parameters.

Property	Coefficient of variation CoV(%)	Source
Unit weight (γ)	3–7%	Harr (1984), Kulhawy(1992)
Effective stress friction angle (ϕ')	2–13%	Harr (1984), Kulhawy(1992)
Undrained shear strength (c_u)	13–40%	Harr (1984), Kulhawy (1992), Duncan and Honorary (2000)

Al-Suhaily (2014) carried out experiments on footings resting on soft clay replaced partially by a trench of granular soil. For the experiments carried out in this study, the reliability analysis will be followed to investigate the effect of uncertainties in soil properties on bearing capacity values. The soft soil in the excavated zone is placed by crushed stone in 2 layers for the 100 mm depth case and 3 layers for 150 mm depth, each layer is 60 mm thick and compacted by using a small hammer to reach the desired dry unit weight of approximately 15.1 (kN/m^3). Square footing (100*100 mm) is used for soil replacement models. Table (2) illustrates details of the soil replacement cases.

Probabilistic Preliminaries

The **probability** of the success of a structure is called its reliability, R, symbolizing the probability of failure as p (f), the standard deviation, a measure of the dispersion of a set of data from its mean (Harr, 2002)**:**

R + p (f) = 1 .... (8)

Table 2: Details of soil replacement models.

Case No.	Type	width of replacement, b (mm)	Depth of replacement, h (mm)
1	square	100	100
2	square	100	150
3	square	200	100
4	square	200	150
5	trench	100	100
6	trench	100	150
7	trench	200	100
8	trench	200	150

Moments

Consider a system of discrete parallel (vertical) forces, P(1), P(2), …, P(N), acting on a rigid beam at the respective distances x(1), x(2), …, x(N), as in Figure (3a). From statics, we have that the magnitude of the equilibrant, M, is:

$M = \sum_{i=1}^{N} P(i)X(i)$ ……. (9)

and its point of application, $\bar{x}$

$\bar{x} = \frac{\sum_{i=1}^{N} P(i)x(i)}{\sum_{i=1}^{N} P(i)}$ ……. (10)

Suppose now that the discrete forces P(i) in Figure (3a) represent the frequencies of the occurrence of the N outcomes x(1), x(2), …, x(N). As the distribution is exhaustive, the magnitude of the equilibrant must be unity, M = 1. Hence, Eq. (10) becomes:

$E[x] = \bar{x} = \sum_{i=1}^{N} P(i)\, x(i)$ ……. (11)

The expected value (mean) provides the locus of the central tendency of the distribution of a random variable. Returning to statics, the measure of the dispersion of the distribution of the force system about the centroid axis, at x = E[x] in Figure (3b), is given by the moment of inertia (the second central moment),

$I(y) = \int_{x(a)}^{x(b)} (x - \bar{x})^2 P(x)\, dx$ ……. (12)

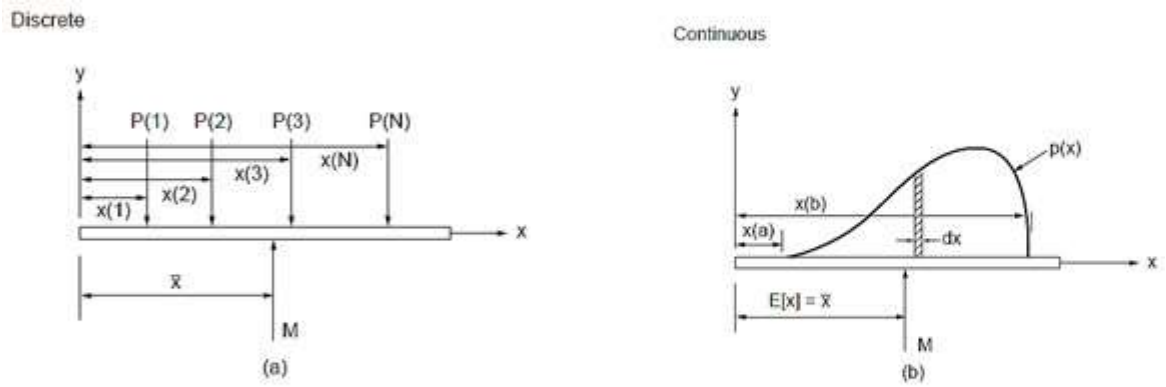

Figure (3) Equilibrant for discrete and continuous distributions, (Harr, 2002).

The equivalent measure of the scatter (variability) of the distribution of a random variable is called its variance, denoted as v[x] and defined as:

Discrete $\boldsymbol{v[x] = \sum_{all\ x(i)} [x(i) - \bar{x}]^2 . P(i)}$ ……. (13)

Continuous $\boldsymbol{v}[\boldsymbol{x}] = \int_{x(a)}^{x(b)} (x-\bar{x})^2 P(x)\,dx$ ……. (14)

In terms of the expectation, these can be written as:

$$\boldsymbol{v}[\boldsymbol{x}] = \boldsymbol{E}[(x-\bar{x})^2] \quad \text{……. (15)}$$

This, after expansion, leads to a form more amenable to computations:

$$\boldsymbol{v}[\boldsymbol{x}] = \boldsymbol{E}[\boldsymbol{x}^2] - (\boldsymbol{E}[\boldsymbol{x}])^2 \quad \text{……. (16)}$$

This expression is the equivalent of the parallel-axis theorem for the moment of inertia. A more meaningful measure of dispersion of a random variable (x) is the positive square root of its variance (compare with radius of gyration of mechanics) called the standard deviation, σ[x], (Harr, 2002):

$$\sigma[\mathrm{x}] = \sqrt{v[x]} \quad \text{……. (17)}$$

It is seen that the standard deviation of the exponential distribution is σ[x] = 1/a. An extremely useful relative measure of the scatter of a random variable (x) is its coefficient of variation CoV(x), usually expressed as a percentage:

$$\mathrm{CoV[x]} = \frac{\sigma[\mathrm{x}]}{\mathrm{E[x]}} * 100\ \% \quad \text{……. (18)}$$

It should be emphasized that a straight line fit can be assumed. The reasonableness of this assumption is provided by the correlation coefficient, P defined as:

$$P = \frac{cov\ [x,y]}{\sigma[x].\sigma[y]} \quad \text{……. (19)}$$

where σ [x], and σ [y] are the respective standard deviations and cov[x, y] is Coefficient of Variation. It is the measure of dispersion of data which is defined as:

$$cov\ [x,y] = \frac{1}{N}\sum_{i=1}^{N}[x(i)-\bar{x}]\,[y(i)-\bar{y}] \quad \text{……. (20)}$$

with analogy to statics, the covariance corresponds to the product of inertia, (Harr, 2002).

Point estimate method — several random variables

Rosenblueth (1975) generalized the methodology for any number of correlated variables. For example, for a function of three random variables say, y = y[x (1), x (2), x (3)], where p (i, j) is the correlation coefficient between variables x(i) and x(j),

$$E[y^N] = P(+++)y^N(+++) + P(++\ -)y^N(++\ -) + \cdots + P(---)y^N(--\ -) \quad \text{……. (21)}$$

where:

$$\mathrm{y}(\pm\pm\pm) = \mathrm{y}[\bar{\mathrm{x}}(1) \pm\ \sigma[\mathrm{x}1], \bar{\mathrm{x}}(2) \pm\ \sigma[\mathrm{x}2], \bar{\mathrm{x}}(3) \pm\ \sigma[\mathrm{x}3]] \quad \text{……. (22)}$$

$$\mathrm{P}(+++) = \mathrm{P}(---) = \frac{1}{2^3}[1+\rho(1,2)+\rho(2,3)+\rho(3,1)] \quad \text{……. (23)}$$

$$P(++\ -) = P(-\ -\ +) = \frac{1}{2^3}[1+\rho(1,2)-\rho(2,3)-\rho(3,1)] \quad \text{……. (24)}$$

$$\mathrm{P}(+\ -+) = \mathrm{P}(-\ +-) = \frac{1}{2^3}[1-\rho(1,2)-\rho(2,3)+\rho(3,1)] \quad \text{……. (25)}$$

$$\mathrm{P}(+\ --) = \mathrm{P}(-\ ++) = \frac{1}{2^3}[1-\rho(1,2)+\rho(2,3)-\rho(3,1)] \quad \text{……. (26)}$$

where σ[xi] is the standard deviation of x (i). The sign of p(i, j) is determined by the multiplication rule of i and j; that is, if the sign of i = (–), and of j = (+), then (i)(j) = (–)(+) = (–). Equation (21) has $2^3 = 8$ terms, all permutations of the three + ves and –ves . In general, for M variables, there are 2^M terms and $M(M-1)/2$ correlation coefficients, the number of combinations of M objects taken two at a time. The coefficient on the right-hand side of Equations (26), in general, is $(1/2)^M$, (Harr, 2002).

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The adequacy of a proposed design in geotechnical engineering is generally determined by comparing the estimated resistance of the system to that of the imposed loading. The resistance is the capacity C (or strength) and the loading is the induced demand D imposed on the structure.

In the procedure presented by Harr (2002) for analysis of footings and developed by Fattah (2010) for piles, a capacity– demand concept will be used. Some common examples are the bearing capacity of a soil and the column loads, allowable and computed maximum stresses, traffic capacity and anticipated traffic flow on a highway, culvert sizes and the quantity of water to be accommodated, and structural capacity and earthquake loads.

Conventionally, the designer forms the well-known factor of safety as the ratio of the single-valued nominal values of capacity *C* and demand *D (*Ellingwood et al., 1980):

$$FS = \frac{C}{D} \qquad \text{……. (27)}$$

In general, the demand function will be the resultant of the many uncertain components of the system under consideration (vehicle loadings, wind loadings, earthquake accelerations, location of the water table, temperatures, quantities of flow, runoff, and stress history, to name only a few). Similarly, the capacity function will depend on the variability of material parameters, testing errors, construction procedures and inspection supervision, ambient conditions, and so on.

If the maximum demand (D_{max}) exceeds the minimum capacity (C_{min}), the distributions overlap (shown shaded), and there is a nonzero probability of failure. The difference between the capacity and demand functions is called the safety margin (S); that is (Harr, 2002):

$$S = C - D \qquad \text{……. (28)}$$

Obviously, the safety margin is itself a random variable. Failure is associated with that portion of its probability distribution wherein it becomes negative (shaded); that portion wherein S = C – D ≤ 0. As the shaded area is the probability of failure P(f), we have:

$$P(f) = P[(C - D) \leq 0] = P[S \leq 0] \qquad \text{……. (29)}$$

The number of standard deviations that the mean value of the safety margin is beyond S = 0, is called the reliability index, β; that is:

$$\beta = \frac{\bar{S}}{\bar{\sigma}[S]} \qquad \text{……. (30)}$$

The reliability index is seen to be the reciprocal of the coefficient of variation of the safety margin, or:

$$\beta = \frac{1}{\text{CoV(S)}} \qquad \text{……. (31)}$$

Application to their definitions produces the following identities (a, b, and c are constants), (Ditlevesen, 1981):

$$E[a + bx + cy] = a + bE[x] + cE[y] \qquad \text{……. (32)}$$

$$v[a + bx + cy = b^2v[x] + c^2v[y] + 2b.c.cov\ [x, y] \qquad \text{……. (33)}$$

$$cov\ [x, y] \leq\ \sigma[x].\sigma[y] \qquad \text{……. (34)}$$

$$v[a + bx + cy = b^2v[x] + c^2v[y] + 2b.c.\sigma[x].\sigma[y].P$$

From Eq. 33, we have:

$$E[S] = E[C] - E[D] = \bar{C} - \bar{D} \qquad \text{……. (35)}$$

Equation 34 produces:

$$\sigma^2[S] = \sigma^2[C] + \sigma^2[D] - 2P\sigma[C].\sigma[D] \qquad \text{……. (36)}$$

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Hence,

$$\beta = \frac{\bar{C}-\bar{D}}{\sqrt{\sigma^2[C]+\sigma^2[D]-2P\sigma[C].\sigma[D]}} \qquad \text{....... (37)}$$

It can be shown that the sum of difference of two normal varieties is also a normal variant (Haugen, 1968). Hence, if it is assumed that the capacity and demand functions are normal variants, it follows that:

$$P(f) = \frac{1}{2} - \psi[\beta] \qquad \text{....... (38)}$$

where $\psi\ [\beta]$ is standard normal probability as given in standard normal probability tables.

Case Study 1:
For the trench of 200 mm wide and 150 mm deep and foundation width of 100 mm, Equation (6) can be used for the estimation of bearing capacity. Table (3) summarizes the parameters adopted in the analysis.

Table 3: The parameters of trench and bed soil used in reliability estimation using Equation 6.

Parameter	Values form experimental work	Standard deviation	X +	X-
Unit weight of crushed stone of the trench (kN/m^3)	15.2	5	20.2	10.2
Angle of friction of crushed stone, ϕ (^{o})	45	5	50	40
Cohesion of the soil bed, c_u (kN/m^2)	17	5.78	22.78	11.22

The bearing capacity is a function of three independent variables, therefore the bearing capacity will be calculated 2^3= 8 times.

Q (ϕ, γ, c) (kN/m^2)	Q^2 (ϕ, γ, c)
Q(+ + +)=523.91	274481.6881
Q(- - -)=143.34	20546.3556
Q(+ - -)=265.69	70591.1761
Q(- + -)=282.06	79557.8436
Q(- - +)=145.14	21065.6196
Q(+ + -)=520.01	270410.4001
Q(- + +)=283.86	80576.4996
Q(+ - +)=269.59	72678.7681

According to Harr (2002), the correlation coefficient p (ϕ, γ, c) = -0.5
Using point estimation method to find the weights p(i, j, k):

$$P(+++) = P(---) = \frac{1}{2^3}[1 + P(1,2) + P(2,3) + P(3,1)] = 0.3125$$

$$P(++\ -) = P(-\ -\ +) = \frac{1}{2^3}[1 + P(1,2) - P(2,3) - P(3,1)] = 0.0625$$

$$P(++\ -) = P(-\ -\ +) = \frac{1}{2^3}[1 + P(1,2) - P(2,3) - P(3,1)] = 0.0625$$

$$P(+\ --) = P(-\ ++) = \frac{1}{2^3}[1 - P(1,2) + P(2,3) - P(3,1)] = 0.0625$$

from Equation (11), finding the mean:

$E[Q] = \bar{Q} = \sum Q(ij)P(ij)$ = 318.91 kN/m^2 while the bearing capacity form the experimental results for the same case was 510 kN/m^2

$E[Q]^2 = \bar{Q} = \sum Q^2(ij)P(ij)$= 129376.28

and from Equation (16) , the variance is :
$\boldsymbol{v}[\mathrm{Q}] = \boldsymbol{E}[\mathrm{Q}^{\mathbf{2}}] - (\boldsymbol{E}[\mathrm{Q}])^{\mathbf{2}}$= 2767.100

and Equation (17) the standard deviation gives:
$\sigma[\mathrm{Q}] = \sqrt{v[Q]}$=166.346
for the coefficient of variation, Equation (20) requires:

$$\mathrm{CoV[Q]} = \frac{\sigma[\mathrm{Q}]}{\mathrm{E[Q]}} * 100\ \% = \frac{166.346}{318.91} * 100 = 52.160\ \%$$

For a factor of safety = 4, the demand will be from Equation (24):

$$\overline{\mathrm{D}} = \frac{\overline{\mathrm{E}}}{\mathrm{F.s}} = 79.728$$

The standard deviation of the demand will be equal to:

$$\sigma[\mathrm{D}] = E(D) * CoV(D) = 79.728 * 0.5216 = 41.586$$

to find the safety margin :

$S = \bar{C} - \bar{D} = 318.91 - 79.728 = 239.184\ \mathrm{kN}/m^2$
Forming the characteristics of the safety margin and according to Harr (2002), the coefficient of correlation will be p(Q, D) = 0.75,: From Equation (37), we have the reliability index:

$$\beta = \frac{\overline{\mathrm{C}} - \overline{\mathrm{D}}}{\sqrt{\sigma^2[\mathrm{C}] + \sigma^2[\mathrm{D}] - 2\mathrm{P}\sigma[\mathrm{C}].\sigma[\mathrm{D}]}}$$

$$\beta = \frac{239.184}{\sqrt{(166.346)^2 + (41.586)^2 - 2(0.75)(166.346)(41.586)}} = 1.734196$$

From probability tables (Appendix C), $\psi\ (\beta\ )$ = 0.4585
From Equation (38):

$$\boldsymbol{P(f)} = \frac{\mathbf{1}}{\mathbf{2}} - \psi(\beta) = \mathbf{0.04144}$$

Probability of failure = 4.14 %

Table (4) shows a summary of the reliability calculations for the trench soil replacement using Equation (6) considering different values of factor of safety.

Case Study 2:
For the same trench, Equation (4) will be used instead of Equation (6) for the estimation of bearing capacity (the problem will be considered as a two-layer soil system). Table (5) summarizes the parameters adopted in the analysis.

The bearing capacity is a function of five independent variables, therefore the bearing capacity will be calculated 2^5=32 times as listed in the table below.

Using point estimation method for five variables to find the weights P(I, j); from equation (11), the mean is calculated as follows:

$E[Q] = \bar{Q} = \sum Q(ij)P(ij)$ = 242.33 kN/m^2 while the bearing capacity form the experimental results for the same case is 510 kN/m^2

$E[Q]^2 = \bar{Q} = \sum Q^2(ij)P(ij)$= 61820.7

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and from Equation (16) , the variance is:
$\boldsymbol{v}[\mathrm{Q}] = \boldsymbol{E}[\mathrm{Q}^{\mathbf{2}}] - (\boldsymbol{E}[\mathrm{Q}])^{\mathbf{2}}$= 3096.69

and Equation (17), the standard deviation gives:
$\sigma[\mathrm{Q}] = \sqrt{v[Q]}$=55.64
for the coefficient of variation, equation (3.17) requires:

$$\mathrm{CoV[Q]} = \frac{\sigma[\mathrm{Q}]}{\mathrm{E[Q]}} * 100\ \% = \frac{55.64}{242.33} * 100 = 22.963\ \%$$

For factor of safety = 4, the demand will be estimated from Equation (3.24):

$$\bar{\mathrm{D}} = \frac{\bar{\mathrm{E}}}{\mathrm{F.s}} = 60.5862$$

The standard deviation of the demand will be equal to:

$$\sigma[\mathrm{D}] = E(D) * CoV(D) = 60.5862 * 0.22963 = 13.911$$

to find the safety margin :

$$S = \bar{C} - \bar{D} = 242.33 - 60.5862 = 181.747\ \mathrm{kN}/m^2$$

Forming the characteristics of the safety margin and according to Harr (2002), the coefficient of correlation will be ρ (Q, D) = 0.75,: From Equation (37), we have the reliability index:

$$\beta = \frac{\bar{\mathrm{C}} - \bar{\mathrm{D}}}{\sqrt{\sigma^2[\mathrm{C}] + \sigma^2[\mathrm{D}] - 2\mathrm{P}\sigma[\mathrm{C}].\sigma[\mathrm{D}]}}$$

$$\beta = \frac{181.747}{\sqrt{(55.64)^2 + (13.911)^2 - 2(0.75)(55.64)(13.911)}} = 3.9387$$

From probability tables, $\psi\ (\beta\ )$ = 0.49996
From equation (38):

$$\boldsymbol{P(f)} = \frac{\mathbf{1}}{\mathbf{2}} - \psi(\beta) = \mathbf{0.00004}$$

Probability of failure =0.004 %
Table (6) shows a summary of the reliability calculations for the trench soil replacement using Equation (4) considering different values of factor of safety.

Table 5: The parameters of trench and bed soil used in reliability estimation using Equation 4.

Parameter	**Values form experimental work**	**Standard deviation**	**X +**	**X-**
Unit weight of crushed stone of the trench (kN/m^3)	15.5	1.5	17	14
Unit weight of soil under the trench (kN/m^3)	16	1.5	17.5	14.5
Angle of friction crushed stone, ϕ (^{o})	40	5	45	35
Angle of friction of soil under the trench, ϕ (^{o})	5	5	10	0
Cohesion of the soil bed, c_u (kN/m^2)	17	5.78	22.78	11.22

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Q (ϕ_1, ϕ_2, γ_1, ,γ_2, c_u) (kN/m^2)		Q^2 (ϕ_1, ϕ_2, γ_1, ,γ_2, c_u)
Q (+,+,+,+,+)=	333.8485392	111454.8471
Q (-,-,-,-,-)=	168.6189858	28432.36237
Q (-,-,-,-,+)=	201.7208071	40691.284
Q (-,-,-,+,-)=	201.7208071	40691.284
Q (-,-,+,-,-)=	228.4273711	52179.06388
Q (-,+,-,-,-)=	168.6189858	28432.36237
Q (+-,-,-,-)=	189.9219913	36070.36278
Q (-,+,+,+,+)=	299.7294511	89837.74387
Q (+,-,+,+,+)=	333.8485392	111454.8471
Q (+,+,-,+,+)=	244.9466943	59998.88304
Q (+,+,+,-,+)=	333.8485392	111454.8471
Q (+,+,+,+,-)=	262.5464592	68930.64325
Q (+,+,-,-,-)=	189.9219913	36070.36278
Q (-,+,+,-,-)=	228.4273711	52179.06388
Q (-,-,+,+,-)=	228.4273711	52179.06388
Q (-,-,-,+,+)=	201.7208071	40691.284
Q (-,-,+,+,+)=	299.7294511	89837.74387
Q (+,-,-,+,+)=	244.9466943	59998.88304
Q (+,+,-,-,+)=	244.9466943	59998.88304
Q (+,+,+,-,-)=	262.5464592	68930.64325
Q (-,+,+,+,-)=	228.4273711	52179.06388
Q (+,-,-,-,+)=	244.9466943	59998.88304
Q (+,-,+,-,+)=	333.8485392	111454.8471
Q (-,+,-,+,-)=	201.7208071	40691.284
Q (+,-,+,-,-)=	262.5464592	68930.64325
Q (-,+,-,+,+)=	201.7208071	40691.284
Q (-,-,+,-,+)=	299.7294511	89837.74387
Q (+,+,-,+,-)=	231.3360494	53516.36777
Q(+,-,-,+,-)=	231.3360494	53516.36777
Q(-,+,+,-,+)=	299.7294511	89837.74387
Q (-,+,-,-,+)=	201.7208071	40691.284
Q (+,-,+,+,-)=	262.5464592	68930.64325

Reliability Relations

Reliability estimation for all cases have been done using the EXECL program and presented in the form of histograms of calculation. The results are shown in Figures 4 to 7. It can be seen that the reliability index increases with the increase of factor of safety while the probability of failure decreases with the increase of factor of safety.

The reliability and the probability of failure depend majorly on the approach used in the estimation of bearing capacity. A number of equations were derived in the literature to estimate the bearing capacity but the degree of conservation for each one is different from one to another.

The reliability analysis helps in choosing the most proper equation for the estimation of bearing capacity which provides adequate factor of safety with a sufficient degree of economy.

In some cases analysed in this study, the probability of failure was found to be less than 1% depending on the value of reliability index obtained from reliability tables which are always greater than 2.2 based on standard normal distribution. This may have to be reanalysed depending on more real values of probability of failure using different types of distribution.

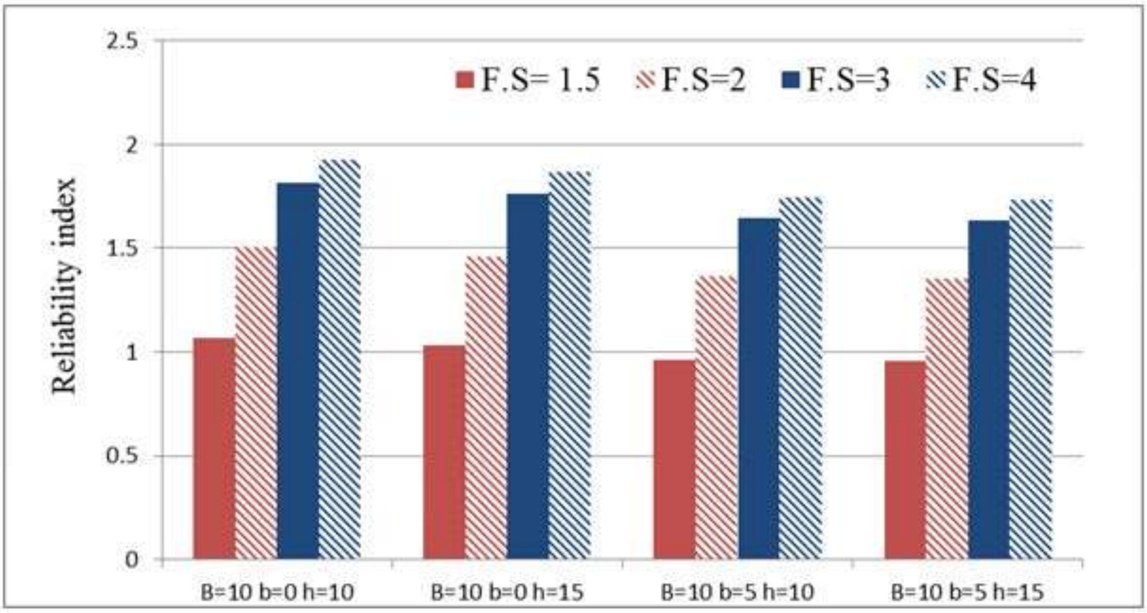

Figure (4) Reliability Index of different cases of a trench of soil replacement considering different values of factor of safety using Equation (6).

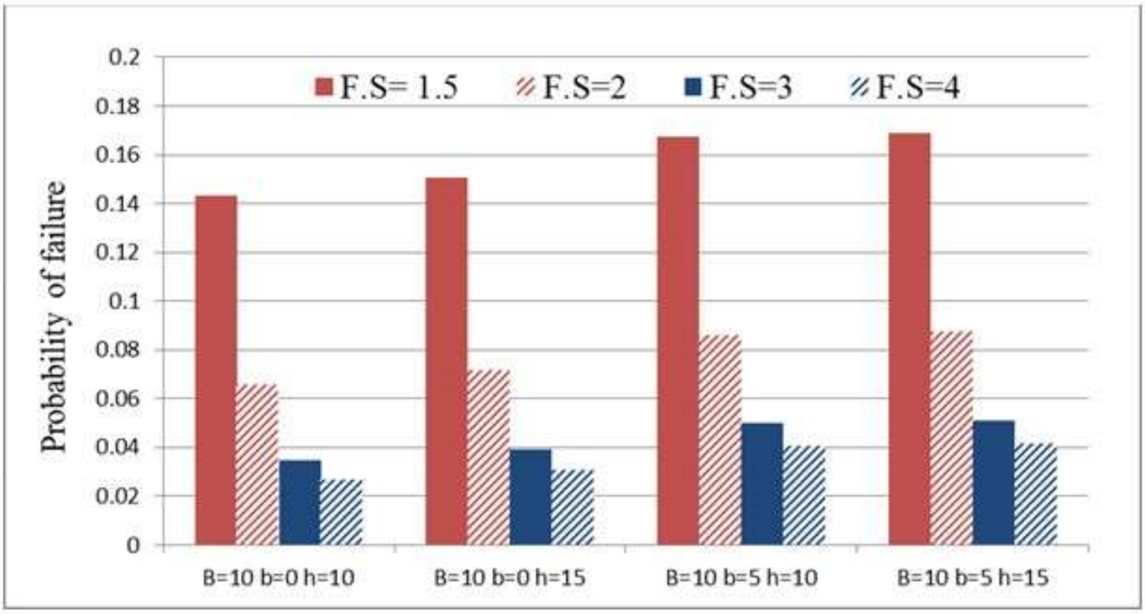

Figure (5) Probability of failure of different cases of a trench of soil replacement considering different values of factor of safety using Equation (6).

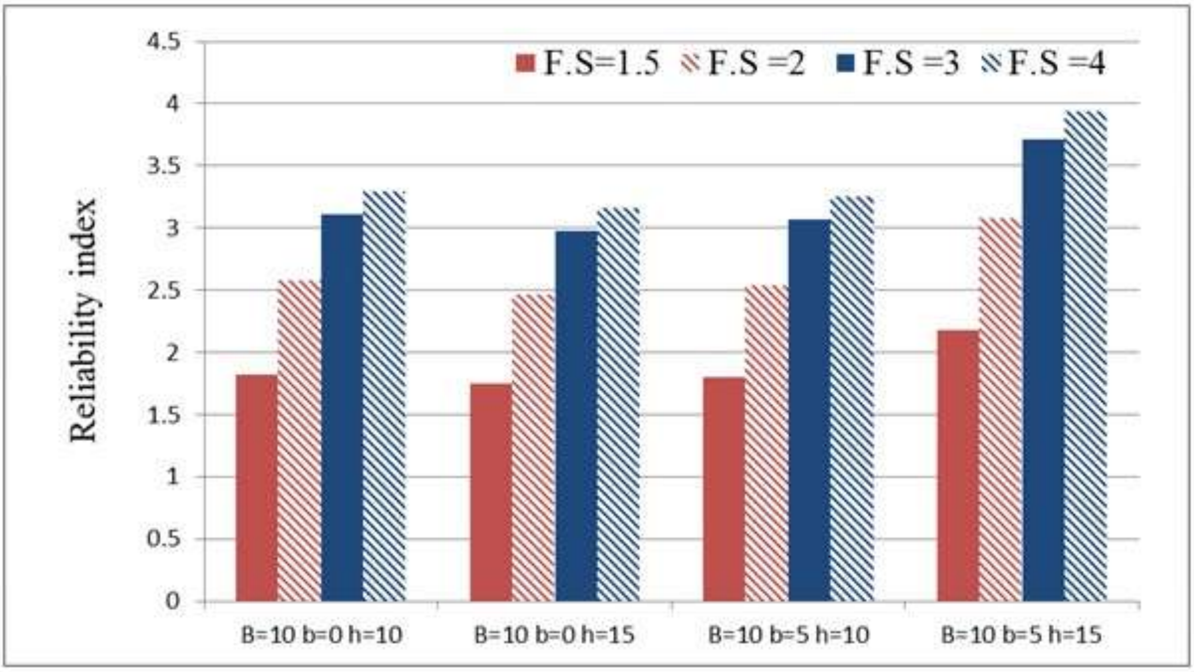

Figure (6) Reliability Index of different cases of trench soil replacement considering different values of factor of safety using Equation (4).

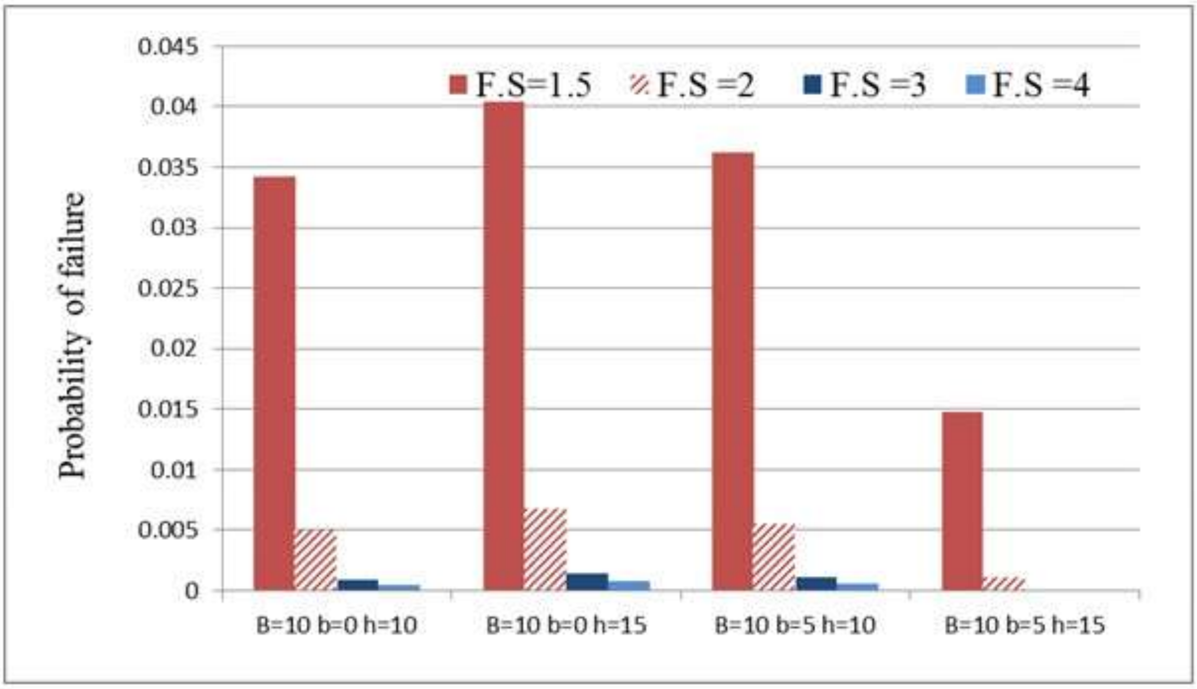

Figure (7) Probability of failure of different cases of a trench of soil replacement considering different values of factor of safety using Equation (4).

CONCLUSIONS

1. The reliability index increases with the increase of factor of safety, while the probability of failure decreases with the increase of factor of safety.
2. The reliability and the probability of failure depend mainly on the approach used in the estimation of bearing capacity, a number of equations were derived in the literature to estimate the bearing capacity but the degree of conservation for each one is different from one to another.

3. In some cases analysed in this study, the probability of failure was found to be less than 1% depending on the value of reliability index obtained from reliability tables which are always greater than 2.2 based on standard normal distribution.
4. The reliability procedure which is an extension of the point estimate method in which the expected values of the standard deviation of the capacity and demand functions are calculated, is found successful. The procedure is adopted using two approaches of estimation of bearing capacity of foundations. In the first approach, the bearing capacity of the trench of replaced soil is considered, while two-layer soil system is considered in the second approach.

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Table 4: Summary of reliability calculations for bearing capacity of different cases of a trench of soil replacement considering different values of factor of safety using equation (6).

Mean	Variance	Standard deviation of capacity	Coefficient of variation	Demand	Standard deviation of Demand	Safety margin	Reliability index	ψ(B)	[illegible]
272.6875	16392.0986	128.0316	46.9518	181.7917	85.3544	90.8958	1.0649	0.3565	0.
297.3375	20676.6773	143.7939	48.3605	198.2250	95.8626	99.1125	1.0339	0.3494	0.
300.9125	24325.5842	155.9666	51.8312	200.6083	103.9777	100.3042	0.9647	0.3326	0.
318.9125	27671.1002	166.3463	52.1605	212.6083	110.8976	106.3042	0.9586	0.3311	0.
272.6875	16392.0986	128.0316	46.9518	136.3438	64.0158	136.3438	1.5060	0.4340	0.
297.3375	20676.6773	143.7939	48.3605	148.6688	71.8969	148.6688	1.4622	0.4282	0.
300.9125	24325.5842	155.9666	51.8312	150.4563	77.9833	150.4563	1.3642	0.4138	0.
318.9125	27671.1002	166.3463	52.1605	159.4563	83.1732	15e9.4563	1.3556	0.4124	0.
272.6875	16392.0986	128.0316	46.9518	90.8958	42.6772	181.7917	1.8163	0.4653	0.
297.3375	20676.6773	143.7939	48.3605	99.1125	47.9313	198.2250	1.7634	0.4611	0.
300.9125	24325.5842	155.9666	51.8312	100.3042	51.9889	200.6083	1.6453	0.4501	0.
318.9125	27671.1002	166.3463	52.1605	106.3042	55.4488	212.6083	1.6350	0.4490	0.
272.6875	16392.0986	128.0316	46.9518	68.1719	32.0079	204.5156	1.9265	0.4730	0.
297.3375	20676.6773	143.7939	48.3605	74.3344	35.9485	223.0031	1.8704	0.4693	0.
300.9125	24325.5842	155.9666	51.8312	75.2281	38.9917	225.6844	1.7452	0.4595	0.
318.9125	27671.1002	166.3463	52.1605	79.7281	41.5866	239.1844	1.7341	0.4586	0.

Table 6: Summary of reliability calculations of different cases of trench soil replacement considering different values of factor of safety using equation (4).

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Mean	Variance	Standard deviation of capacity	Coefficient of variation	Demand	Standard deviation of Demand	Safety margin	Reliability index	ψ(B)	P(f
173.6063	2268.617	47.63000	27.43564	115.73752	31.75333	57.86876	1.82245	0.46581	0.034
255.1469	5337.94	73.06121	28.63496	170.09791	48.70747	85.04895	1.74612	0.45960	0.040
171.1198	2267.583	47.61915	27.82796	114.07987	31.74610	57.03994	1.79675	0.46381	0.036
242.3305	3096.696	55.64796	22.96367	161.55364	37.09864	80.77682	2.17735	0.48527	0.014
173.6063	2268.617	47.63000	27.43564	86.80314	23.81500	86.80314	2.57733	0.49502	0.004
255.1469	5337.94	73.06121	28.63496	127.57343	36.53061	127.57343	2.46938	0.49323	0.006
171.1198	2267.583	47.61915	27.82796	85.55990	23.80957	85.55990	2.54099	0.49447	0.005
242.3305	3096.696	55.64796	22.96367	121.16523	27.82398	121.16523	3.07924	0.49896	0.001
173.6063	2268.617	47.63000	27.43564	57.86876	15.87667	115.73752	3.10838	0.49906	0.000
255.1469	5337.94	73.06121	28.63496	85.04895	24.35374	170.09791	2.97819	0.49855	0.001
171.1198	2267.583	47.61915	27.82796	57.03994	15.87305	114.07987	3.06455	0.49891	0.001
242.3305	3096.696	55.64796	22.96367	80.77682	18.54932	161.55364	3.71370	0.49990	0.000
173.6063	2268.617	47.63000	27.43564	43.40157	11.90750	130.20471	3.29693	0.49951	0.000
255.1469	5337.94	73.06121	28.63496	63.78672	18.26530	191.36015	3.15884	0.49921	0.000
171.1198	2267.583	47.61915	27.82796	42.77995	11.90479	128.33985	3.25045	0.49942	0.000
242.3305	3096.696	55.64796	22.96367	60.58262	13.91199	181.74785	3.93898	0.49996	0.000

Appendix

The bearing capacity factors of the granular trench

Figures (I), (II) and (III) show the bearing capacity factors for granular trench according to Madhav and Vitkar's (1974).

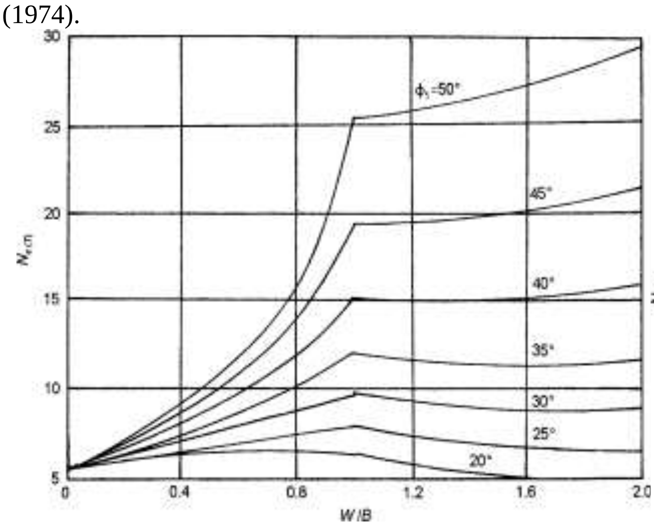

Figure (I) Madhav and Vitkar's bearing capacity factor, *Nc* (*T*).

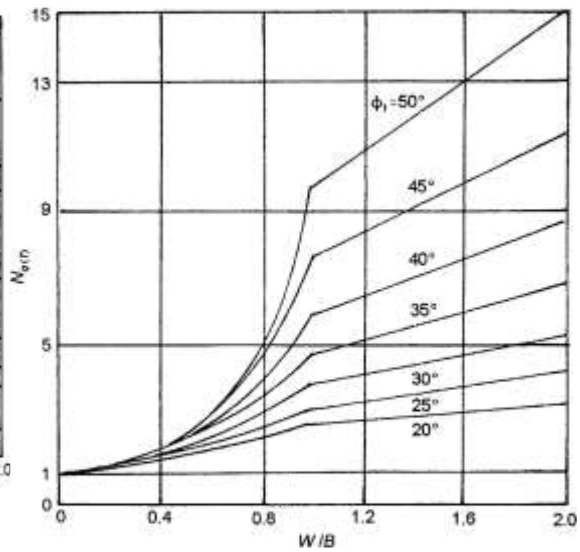

Figure (II) Madhav and Vitkar's bearing capacity factor, *Nq* (*T*).

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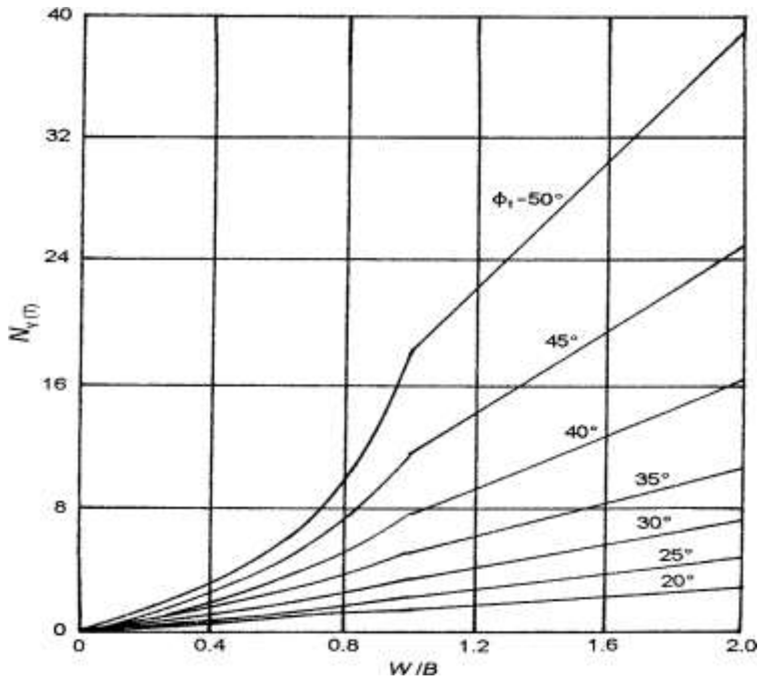

Figure (III) Madhav and Vitkar's bearing capacity factor, $N\square$ (T).

Probability Table

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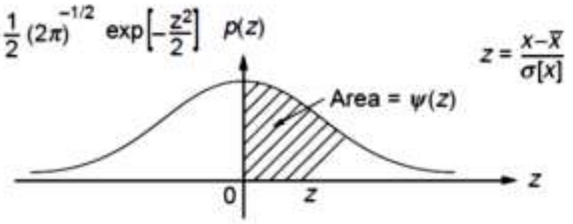
for $z > 2.2$, $\psi(z) \approx \frac{1}{2} - \frac{1}{2}(2\pi)^{-1/2} \exp\left[-\frac{z^2}{2}\right]$



z	0	1	2	3	4	5	6	7	8	9
0	0	.003969	.007978	.011966	.015953	.019939	.023922	.027903	.031881	.035856
.1	.039828	.043795	.047758	.051717	.055670	.059618	.063559	.067495	.071424	.075345
.2	.079260	.083166	.087064	.090954	.094835	.098706	.102568	.106420	.110251	.114092
.3	.117911	.121720	.125516	.129300	.133072	.136831	.140576	.144309	.148027	.151732
.4	.155422	.159097	.162757	.166402	.170031	.173645	.177242	.180822	.184386	.187933
.5	.191462	.194974	.198466	.201944	.205401	.208840	.212260	.215661	.219043	.222405
.6	.225747	.229069	.232371	.235653	.238914	.242154	.245373	.248571	.251748	.254903
.7	.258036	.261148	.264238	.267305	.270350	.273373	.276373	.279350	.282305	.285236
.8	.288145	.291030	.293892	.296731	.299546	.302337	.305105	.307850	.310570	.313267
.9	.315940	.318589	.321214	.323814	.326391	.328944	.331472	.333977	.336457	.338913
1.0	.341345	.343752	.346136	.348495	.350830	.353141	.355428	.357690	.359929	.362143
1.1	.364334	.366500	.368643	.370762	.372857	.374928	.376976	.379000	.381000	.382977
1.2	.384930	.386861	.388768	.390651	.392512	.394350	.396165	.397958	.399727	.401475
1.3	.403200	.404902	.406582	.408241	.409877	.411492	.413085	.414657	.416207	.417736
1.4	.419243	.420730	.422196	.423641	.425066	.426471	.427855	.429219	.430563	.431888
1.5	.433193	.434476	.435745	.436992	.438220	.439429	.440620	.441792	.442947	.444083
1.6	.445201	.446301	.447384	.448449	.449497	.450529	.451543	.452540	.453521	.454486
1.7	.455435	.456367	.457284	.458185	.459070	.459941	.460796	.461636	.462462	.463273
1.8	.464070	.464852	.465620	.466375	.467116	.467843	.468557	.469258	.469946	.470621
1.9	.471283	.471933	.472571	.473197	.473610	.474412	.475002	.475581	.476148	.476705
2.0	.477250	.477784	.478308	.478822	.479325	.479818	.480301	.480774	.481237	.481691
2.1	.482136	.482571	.482997	.483414	.483823	.484222	.484614	.484997	.485371	.485738
2.2	.486097	.486447	.486791	.487126	.487455	.487776	.488089	.488396	.488696	.488989
2.3	.489276	.489556	.489830	.490097	.490358	.490613	.490863	.491106	.491344	.491576
2.4	.491802	.492024	.492240	.492451	.492656	.492857	.493053	.493244	.493431	.493613
2.5	.493790	.493963	.494132	.494297	.494457	.494614	.494766	.494915	.495060	.495201
2.6	.495339	.495473	.495604	.495731	.495855	.495975	.496093	.496207	.496319	.496427
2.7	.496533	.496636	.496736	.496833	.496928	.497020	.497110	.497197	.497282	.497365
2.8	.497445	.497523	.497599	.497673	.497744	.497814	.497882	.497948	.498012	.498074
2.9	.498134	.498193	.498250	.498305	.498359	.498411	.498462	.498511	.498559	.498605
3.0	.498650	.498694	.498736	.498777	.498817	.498856	.498893	.498930	.498965	.498999
3.1	.499032	.499065	.499096	.499126	.499155	.499184	.499211	.499238	.499264	.499289
3.2	.499313	.499336	.499359	.499381	.499402	.499423	.499443	.499462	.499481	.499499
3.3	.499517	.499534	.499550	.499566	.499581	.499596	.499610	.499624	.499638	.499651
3.4	.499663	.499675	.499687	.499698	.499709	.499720	.499730	.499740	.499749	.499758
3.5	.499767	.499776	.499784	.499792	.499800	.499807	.499815	.499822	.499828	.499835
3.6	.499841	.499847	.499853	.499858	.499864	.499869	.499874	.499879	.499883	.499888
3.7	.499892	.499896	.499900	.499904	.499908	.499912	.499915	.499918	.499922	.499925
3.8	.499928	.499931	.499933	.499936	.499938	.499941	.499943	.499946	.499948	.499950
3.9	.499952	.499954	.499956	.499958	.499959	.499961	.499963	.499964	.499966	.499967